Legal Contractions: A Logical Analysis

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ABSTRACT

This paper systematically investigates how to model legal contraction in an expressive variant of Defeasible Deontic Logic. We argue that legal contraction is an umbrella concept that includes operations which are conceptually and technically different: removing rules, adding exceptions, and modifying rule priorities. The peculiarities of deleting legal conclusions show that an extension of those operations is sometimes needed, which works on the indirect conclusions from which the target effect of the contraction is obtained. The proposed techniques are discussed in the context of a new version for the logic of AGM postulates of contraction.

Categories and Subject Descriptors

I.2.3 [**Deduction and Theorem Proving**]: Nonmonotonic reasoning and belief revision

Keywords

Norm change; AGM; Legal reasoning

1. INTRODUCTION

One peculiar feature of the law is that it necessarily takes the form of a dynamic normative system [20, 19]. Despite the importance of norm-change mechanisms, only a few formal models of law dynamics are proposed in the literature (for a concise review, see [18]). First of all, in modeling the dynamics of law it is essential to distinguish norms from obligations and permissions [7, 12]: the latter ones are just possible effects of the application of norms and their dynamics do not necessarily require to remove or revise norms, but correspond in most cases to instances of the notion of *norm defeasibility* [12]. In fact, obligations can change with the normative system being the same. For example, due to change in the world, new obligations can be detached from the legal norms. Hence, the mechanisms of how norms change are different from those of

how obligations change. How these two aspects of legal dynamics logically interplay is still an open research issue. We focus on the issue of norm-change.

To the best of our knowledge, Alchourrón and Makinson were the first to study the changes of a legal code [3, 4, 2]. The addition of a new norm *n* causes an enlargement of the code, consisting of the new norm plus all the regulations that can be derived from *n*. Alchourrón and Makinson distinguish two other types of change. When the new norm is incoherent with the existing ones, we have an *amendment* of the code: in order to coherently add the new regulation, we need to reject those norms that conflict with *n*. Finally, *derogation* is the elimination of a norm *n* together with whatever part of the legal code that implies *n*.

Alchourrón, Gärdenfors and Makinson inspired by the works above proposed the so called AGM framework for belief revision [1]. This area proved to a very fertile one and the phenomenon of revision of logical theories has been thoroughly investigated. It is then natural to ask if belief revision offers a satisfactory framework for the problem of norm revision. Some of the AGM axioms seem to be rational requirements in a legal context, whereas they have been criticized when imposed on belief change operators. An example is the *success* postulate, requiring that a new input must always be accepted in the belief set. It is reasonable to impose such a requirement when we wish to enforce a new norm or obligation. However, it gives rise to irrational behaviors when imposed to a belief set, as observed in [9].

As is well-known, the AGM framework distinguishes three types of change operation over theories. *Contraction* is an operation that removes a specified sentence ϕ from a given theory Γ (a logically closed set of sentences) in such a way that Γ is set aside in favour of another theory Γ_ϕ^- which is a subset of Γ not containing ϕ. *Expansion* operation adds ϕ to Γ so that the resulting theory Γ_ϕ^+ is the smallest logically closed set that contains both Γ and ϕ. *Revision* operation adds ϕ to Γ but it is ensured that the resulting theory Γ_ϕ^* is consistent [1]. Alchourrón, Gärdenfors and Makinson argued that, when Γ is a code of legal norms, contraction corresponds to norm derogation (norm removal) and revision to norm amendment.

The operation of contraction is perhaps the most controversial one, due to the elusive nature of legal changes such as derogations and repeals, which are all meant to contract legal effects but in remarkably different ways [12]. Standard AGM framework is of little help here: it has the advantage of being very abstract—it works with theories consisting of simple logical assertions—but precisely for this reason it is more suitable to capture the dynamics of obligations and permissions than the one of legal norms. In fact, it is hard in AGM to represent how the same set of legal effects can be contracted in many different ways, depending on how norms are changed. For this reason, previous works [13, 14, 12] proposed to

combine a rule-based system like Defeasible Logic [6] with some forms of temporal reasoning. The resulting logics had the purpose to faithfully mirror legal changes, thus devising very complex formalisms that (i.) work with at least three distinct time lines (time of legal effects, time of force of norms, time of the legal system), and (ii.) reason about meta-rules, i.e., nested rules. Although no computational results have been given for these contributions, we may reasonably expect that the well-known feasibility of Defeasible Logic is not there preserved.

Recently, this difficulty has been considered and some research has been carried out to reframe AGM ideas within reasonably richer rule-based logical systems able to capture the distinction between norms and legal effects [25, 23]. However, these attempts suffer from some drawbacks: they fail to handle reasoning on deontic effects and are based on a very simple representation of legal systems.

In this paper, we work with a quite expressive logic and systematically investigate three basic and different AGM-like operations of changing legal systems in order to contract legal effects (rule removal, adding exceptions, revising priorities); two of them were previously identified in [12, 23] but were criticized and never fully formalized: in this paper we fill the gap by adding a third type of contraction, avoid some of those critiques and identify subtypes for each of those operations. We also study the peculiarities of legal contraction by distinguishing two components of legal systems: norms defining legal concepts (counts-as rules) and norms stating deontic effects.

The layout of the paper is as follows. Section 2 introduces the logic to model norms and legal effects. In Section 3, we propose three distinct methodologies to change a normative theory, while Section 4 generalises them when it is not possible to work on specific rules. Section 5 explains to what extent our framework joins the AGM postulates for contraction. Section 6 concludes the work.

2. LOGIC

The following is a modal extension of DL, which slightly revises earlier works of [11, 10]. Since the present framework reasons about legal contexts, two kinds of rules are used: *counts-as* (or *constitutive*) *rules*, and *obligation* (or *regulative*) *rules*. Counts-as rules define the terms and concepts of normative systems [24]. These terms and concepts are used to define the condition under which obligation rules (i.e., rules describing the ideal or legal behavior) are applicable. Obligation rules are meant to devise suitable logical conditions for introducing deontic modalities. For example, while a counts-as rule such as $a_1,\ldots,a_n \Rightarrow_c b$ supports the conclusion of b,[1] rules such as $a_1,\ldots,a_n \Rightarrow_O b$ allow to derive $O\,b$, meaning that b is obligatory.

The basic language is defined as follows. Given a set PROP of *propositional atoms*, the set of *literals* Lit is the set of such atoms and their negation. If q is a literal, $\sim q$ denotes the *complementary* literal; if q is a positive literal p then $\sim q$ is $\neg p$, and if q is $\neg p$, then $\sim q$ is p. If MOD = $\{O\}$, the set ModLit of *modal literals* is defined as follows: (i) if $X \in$ MOD and $l \in$ Lit then Xl and $\neg Xl$ are modal literals; (ii) nothing else is a modal literal.

Given Lbl a set of arbitrary labels, every rule in our framework is of the type $r : A(r) \hookrightarrow_X \psi$, where

1. $r \in$ Lbl is the *name* of the rule;
2. $A(r) = \{\phi_1,\ldots,\phi_n\}$, the *antecedent* (or *body*) of the rule, is a finite set denoting the premises of the rule. If r is a counts-as rule, then each ϕ_i belongs to Lit, otherwise it belongs to Lit ∪ ModLit;

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[1] From now on, we omit the mode c attached to a literal derived by a counts-as rule; this choice is justified by the role of counts-as rules, i.e., to define or classify concepts in the domain of interest.

3. $\hookrightarrow \in \{\to, \Rightarrow, \rightsquigarrow\}$ denotes the *type* of the rule;
4. $X \in \{c, O\}$ represents the *mode* of the conclusion;
5. $\psi \in$ Lit is the *consequent* (or *head*) of the rule.

The intuition behind the different arrows is the following. *Strict rules* have the form $\phi_1,\ldots,\phi_n \to_X \psi$. *Defeasible rules* have the form $\phi_1,\ldots,\phi_n \Rightarrow_X \psi$. A rule of the form $\phi_1,\ldots,\phi_n \rightsquigarrow_X \psi$ is a *defeater*. The three types of rules establish the strength of the relationship. Strict rules provide the strongest connection between a set of premises and their conclusion: whenever the premises are deemed as indisputable, so is the conclusion. Defeasible rules allow to derive the conclusion unless there is evidence for its contrary. Finally, defeaters suggest that there is a connection between its premises and the conclusion, but not strong enough to warrant the conclusion on its own; they are meant to defeat rules for the opposite conclusion.

Several abbreviations are used on sets of rules. For example, $R[p]$ (resp. $R^X[p]$ with $X \in \{c, O\}$) denotes the set of all rules whose consequent is p (resp. with type X). The sets R_s, R_d, R_{sd}, and R_{dft} denote the set of strict rules, defeasible rules, strict and defeasible rules, and defeaters respectively.

A *normative theory* is the knowledge base used to reason about the applicability of legal rules included in the theory itself.

DEFINITION 1 (NORMATIVE THEORY). *A normative theory is a structure $D = (F, R^c, R^O, \succ)$, where*

- *$F \subseteq$ Lit ∪ ModLit is a finite set of* facts*;*
- *R^c, R^O are finite sets of counts-as and obligation rules, respectively;*
- *$\succ$ is an acyclic binary relation (called* superiority relation*) defined over $(R^c \times R^c) \cup (R^O \times R^O)$.*

Facts in a normative theory are indisputable statements that are deemed always to be true, while the *superiority relation* determines the relative strength of *conflicting* rules (i.e., rules with complementary heads) with the same mode, in case both rules apply in a particular context.

A *proof P of length n in a normative theory D* is a finite sequence $P(1),\ldots,P(n)$ of *tagged literals* of the type $+\Delta^X q$, $-\Delta^X q$, $+\partial^X q$ and $-\partial^X q$. The proof conditions below define the logical meaning of such tagged literals. Given a normative theory D, $+\Delta^X q$ means that literal q is provable in D with mode X using only facts and strict rules, $-\Delta^X q$ that it has been proved in D that q is not definitely provable in D with mode X, $+\partial^X q$ that q is defeasibly provable in D with mode X, and $-\partial^X q$ that it has been proved in D that q is not defeasibly provable in D with mode X.[2] Given $p \in$ Lit, we say that $D \vdash \pm\#^X p$ iff there exists a proof P of length n in D such that $P(n) = \pm\#^X p$, for $\# \in \{\Delta, \partial\}$.

We now show the proof conditions for strict/defeasible provability/refutability. As a conventional notation, $P(1..n)$ denotes the initial part of proof P of length n.

If $X \in \{c, O\}$, the definition of $+\Delta^X$ describes forward (monotonic) chaining of strict rules:

$$+\Delta^X: \text{If } P(n+1) = +\Delta^X q \text{ then either}$$
$$(1)\ q \in F \text{ if } X = c \text{ or } Xq \in F, \text{ or}$$
$$(2)\ \exists r \in R_s^X[q]:\ \forall a, Xb, \neg Yd \in A(r),$$
$$+\Delta^c a, +\Delta^X b, -\Delta^Y d \in P(1..n).$$

Conditions for the negative counterpart are obtained by using the principle of *strong negation* of [5] and [15]. The tag $-\Delta^X$ states that it is not possible to obtain a conclusion by only using forward chaining of facts and strict rules.

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[2] Henceforth, we use terms *refuted* and *not provable* as synonyms.

$-\Delta^X$: If $P(n+1) = -\Delta^X q$ then
(1) $q \notin F$ if $X = c$ or $Xq \notin F$, and
(2) $\forall r \in R_s^X[q] : \exists a$ or $\exists Xb$ or $\exists\neg Yd \in A(r)$,
$-\Delta^c a$ or $-\Delta^X b$ or $+\Delta^Y d \in P(1..n)$.

To show that a literal q is defeasibly provable with the mode X we have two choices: (1) we show that q is already definitely provable, or (2) we need to argue using the defeasible part of the theory. In the second case, some conditions must be satisfied. First, we need to consider possible reasoning chains in support of $\sim q$ with mode X, and show that $\sim q$ is not definitely provable with that mode (2.1 below). Second, we require that there must be a strict or defeasible rule with mode X for q which can apply (2.2 below). Third, we must consider the set of all rules which are not known to be inapplicable and which permit to get $\sim q$ with mode X (2.3 below). Essentially, each rule s of this kind attacks the conclusion q. To prove q, s must be counterattacked by a rule t for q with the following properties (2.3.2 below): (i) t must be applicable, and (ii) t must prevail over s. Thus, each attack on the conclusion q must be counterattacked by a stronger rule. In other words, r and the rules like t form a team (for q) that defeats the rule s.

$+\partial^X$: If $P(n+1) = +\partial^X q$ then
(1) $+\Delta^X q \in P(1..n)$ or
(2) (2.1) $-\Delta^X \sim q \in P(1..n)$ and
(2.2) $\exists r \in R_{sd}^X[q] : \forall a, Xb, \neg Yd \in A(r)$,
$+\partial^c a, +\partial^X b, -\partial^Y d \in P(1..n)$, and
(2.3) $\forall s \in R^X[\sim q]$ either
(2.3.1) $\exists a$ or $\exists Xb$ or $\exists\neg Yd \in A(s)$,
$-\partial^c a$ or $-\partial^X b$ or $+\partial^Y d \in P(1..n)$, or
(2.3.2) $\exists t \in R^X[q]: \forall a, Xb, \neg Yd \in A(t)$,
$+\partial^c a, +\partial^X b, -\partial^Y d \in P(1..n)$
and $t \succ s$.

Once again, the negative tag for defeasible conclusions derives by applying the principle of strong negation.

$-\partial^X$: If $P(n+1) = -\partial^X q$ then
(1) $-\Delta^X q \in P(1..n)$ and
(2) (2.1) $+\Delta^X \sim q \in P(1..n)$ or
(2.2) $\forall r \in R_{sd}^X[q] : \exists a$ or $\exists Xb$ or $\exists\neg Yd \in A(r)$,
$-\partial^c a$ or $-\partial^X b$ or $+\partial^Y d \in P(1..n)$, or
(2.3) $\exists s \in R^X[\sim q]$:
(2.3.1) $\forall a, Xb, \neg Yd \in A(s)$,
$+\partial^c a, +\partial^X b,$ or $-\partial^Y d \in P(1..n)$; and
(2.3.2) $\forall t \in R^X[q]: \exists a$ or $\exists Xb$ or $\exists\neg Yd \in A(t)$,
$-\partial^c a$ or $-\partial^X b$ or $+\partial^Y d \in P(1..n)$,
or $t \not\succ s$.

Given a normative theory D, the *universe* of D (U^D) is the set of all the atoms occurring in D. Given $\# \in \{\Delta, \partial\}$, the *extension* E^D of D is a structure $(\Delta^+(D), \Delta^-(D), \partial^+(D), \partial^-(D))$:

$\#^+(D) = \{l : D \vdash +\#^c l\} \cup \{Ol : D \vdash +\#^O l\} \cup \{\neg Ol : D \vdash -\#^O l\}$;

$\#^-(D) = \{l : D \vdash -\#^c l\} \cup \{Ol : D \vdash -\#^{Ol}\} \cup \{\neg Ol : D \vdash +\#^O l\}$.

We use the shortcut forms $\Delta^\pm$, $\partial^\pm$ when D is clear from the context.

The positive extension of a theory contains the formulas that are provable in the theory. Notice that we stipulate that $\neg Ol$ is provable when we have $-\partial^O l$, in other terms, we can prove it when we have a refutation of Ol.

In DL, there may be different ways to prove a given literal l: we may have many rules for it, and an analogous situation may occur for each antecedent of one of such rules. When we consider only one rule at a time for each step from l back on, we are looking at a *reasoning chain* for l.

DEFINITION 2 (REASONING CHAIN). *Let* $D = (F, R^c, R^O, \succ)$ *be a normative theory,* $\# \in \{\Delta, \partial\}$ *and* $p \in \text{Lit} \cup \text{ModLit}$. *A* $\#^X$*-chain* $\mathscr{C}[p]$ *in D for p is a finite sequence* $\mathscr{R}_1, \ldots, \mathscr{R}_n$ *where*

- $\forall i \in \{1, \ldots, n\}$, *if $X = c$ then* $\mathscr{R}_i \subseteq R^c$, *otherwise* $\mathscr{R}_i \subseteq R^c \cup R^O$;
- $\mathscr{R}_n \subseteq \begin{cases} R^c[p] & p \in \text{Lit} \\ R^O[q] & p = Oq \\ R^O[\sim q] & p = \neg Oq \end{cases}$
- $\forall k \in \{1, \ldots, n\}$, $\forall r^k \in \mathscr{R}_k$, $\forall d \in A(r^k)$,

$$\exists r' \in \mathscr{R}_{k-1}: \begin{cases} r' \in R_{sd}^c[d] & d \in \text{Lit}, R^c[d] \neq \emptyset \\ r' \in R_{sd}^O[q] & d = Oq, R^O[q] \neq \emptyset \\ r' \in R^O[\sim q] & d = \neg Oq, R_{sd}^O[q] \neq \emptyset \end{cases}$$

- $\forall k \in \{1, \ldots, n-1\}$, $\forall r^k \in \mathscr{R}_k, \exists r^{k+1} \in \mathscr{R}_{k+1}$ *such that either*
  - $C(r^k) \in A(r^{k+1})$, *or*
  - $OC(r^k) \in A(r^{k+1})$, *or*
  - $\neg O \sim C(r^k) \in A(r^{k+1})$.

For all $s \in \mathscr{R}_k$, $1 \leq k \leq n$, *we say that s is in $\mathscr{C}$. If a literal l occurs in the head or the body of any s in $\mathscr{C}$, then l is in $\mathscr{C}$.*

When all rules in a $\#^X$*-chain $\mathscr{C}$ are strict, then $\mathscr{C}$ is a strict chain and $\# = \Delta$; otherwise, $\# = \partial$ and $\mathscr{C}$ is a defeasible chain.*

If $\mathscr{C} = \mathscr{R}_1, \ldots, \mathscr{R}_n$, *the following set is called the* root *of $\mathscr{C}$:*

$$root(\mathscr{C}) = \{d | d \in A(r) : r \in \mathscr{R}_k, 1 \leq k \leq n, \forall r^{k-1} \in \mathscr{R}_{k-1},$$
$$d \neq C(r^{k-1})\}.$$

In this process, we consider defeaters only when we work on negative conclusions such as $-\partial^O p$ (which means that $\neg Op$ is provable) since their purpose is to prevent conclusions and not to directly support them.

DEFINITION 3 (ACTIVE AND INACTIVE CHAIN). *Let D be a normative theory.*

1. *A* $\#^X$*-chain $\mathscr{C}[p]$ in D is* active *in D iff for each literal $a, Xb, \neg Yd$ in $\mathscr{C}[p]$, $D \vdash +\partial_c a$, $D \vdash +\partial_X b$, and $D \vdash -\partial_Y d$;*
2. *A* $\#^X$*-chain $\mathscr{C}[p]$ in D is* inactive *in D iff there is an a, Xb, or $\neg Yd$ in $\mathscr{C}[p]$, such that $D \vdash -\partial_c a$, $D \vdash -\partial_X b$, or $D \vdash +\partial_Y d$.*

3. BASIC LEGAL CONTRACTION

3.1 Contraction by Rule Removal

When our purpose is to contract a legal effect p that follows from a normative theory D, one option is removing rules in D that directly prove p.

REMARK 1. *If legal norms are modeled using rules, the contraction of legal effects by means of rule removal intuitively corresponds to law-making acts, such as the legislative or judicial repeal of statutes and, in common-law systems, judicial overruling (assuming, of course, a rule-based representation of precedents).*

Consider the following example of judicial repeal of a statute (adapted from the case-law of the Italian Constitutional Court):

> *On account of Art. 3 of the Constitution [. . .] the Constitutional Court hereby declares the constitutional illegitimacy of Art. 1 of the Legislative Act n. 124, 23 July 2008 [. . .]*

being that art. 3 of the Italian Constitution reads

> *Article 3*
> *(1) All citizens have equal social status and are equal before the law [. . .].*

while art. 1 of the Legislative Act n. 124, 23 July 2008 states

The Constitutional Court repeals art. 1 above by removing it and its legal effects from the Italian legal system.

Rule removal is captured by the following definitions.

DEFINITION 4. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $D \vdash +\Delta^Y p$ with $Y \in \{c, O\}$. We define $D_p^{-\Delta} = (F, R'^c, R'^O, \succ)$ to be the minimal* strict removal contraction theory *of D by p such that*

1. $R_s^c = R_s^c \setminus \{r \in R_s^c[p] : A(r) \subseteq \Delta^+\}$ *if* $p \in \mathrm{Lit}$
2. $R_s^O = R_s^O \setminus \{r \in R_s^O[q] : A(r) \subseteq \Delta^+\}$ *if* $p = Oq$
3. $R = R \cup \{r' :\to_O q\}$ *if* $p = \neg Oq$.

Definition 4 removes all strict applicable rules for the literal to be contracted in a given theory. However, if the literal to contract is a negative obligation[3], then the contraction is *de facto* an expansion for the opposite literal.

DEFINITION 5. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $D \vdash +\partial^Y p$ with $Y \in \{c, O\}$. We define $D_p^{-\partial} = (F, R'^c, R'^O, \succ')$ to be the minimal* defeasible removal contraction theory *of D by p such that*

1. $R \setminus \{r \in R_{sd}^c[p] : A(r) \subseteq \partial^+ \text{ and } \{s \in R^c[\sim p] : s \succ r, A(s) \subseteq \partial^+\} = \emptyset\}$ *if* $p \in \mathrm{Lit}$
2. $R \setminus \{r \in R_{sd}^O[q] : A(r) \subseteq \partial^+ \text{ and } \{s \in R^O[\sim q] : s \succ r, A(s) \subseteq \partial^+\} = \emptyset\}$ *if* $p = Oq$
3. $R \setminus \{r \in R^O[\sim q] : A(r) \subseteq \partial^+ \text{ and } \{s \in R^O[q] : s \succ r, A(s) \subseteq \partial^+\} = \emptyset\}$ *if* $p = \neg Oq$ *and* $\{t \in R_{sd}^O[q] : A(t) \subseteq \partial^+\} \neq \emptyset$
4. $R \cup \{r' :\Rightarrow_O q\}$ *and* $\succ \cup \{r' \succ s : s \in R^O[\sim q], A(s) \subseteq \partial^+\}$ *if* $p = \neg Oq$ *and* $\{t \in R_{sd}^O[q] : A(t) \subseteq \partial^+\} = \emptyset$.

Definition 5 acts analogously on defeasible rules, with some notable differences. First of all, to preserve the minimality of the contraction we have to identify all rule essential to prove the literal to be contracted. According to [21] we have to remove only the applicable rules that are not inferiorly defeated (i.e., the set of rules such that have no stronger applicable rules). The second difference concerns the contraction of negative obligations. We have to consider that there might be applicable and undefeated rules for the opposite. In such case, the contraction works by removing such rules. These operations are illustrated in Example 1.

EXAMPLE 1. *Let $D = (\{a, b, c\}, \emptyset, R^O, \{(r_3, r_2)\})$, where R^O is*

$$
\begin{array}{llll}
r_1: & a & \Rightarrow_O & \sim q \\
r_2: & b & \Rightarrow_O & \sim q \\
r_3: & c & \Rightarrow_O & q.
\end{array}
$$

Condition 3: the removal of r_1 allows the defeasible rule-contraction theory $D_{\neg Oq}^{-\partial} \vdash +\partial_O q$ since $r_3 \succ r_2$.
Condition 4: if we suppose $c \notin F$, then r_3 is not applicable and a new rule $r' :\Rightarrow_O q$ is added to R^O which is stronger than r_1 and r_2.

As mentioned above, to contract a negative obligation (e.g., $\neg Op$) sometimes we have to remove rules, while in other cases we have to introduce a new one. While this might appear to be a drawback on the overall approach and an ad hoc solution to deal with some oddities of the underlying modeling formalism, we argue that this

is grounded in legal theory. Indeed the handling of these two cases depends on the meaning of a negative obligation $\neg O$. Remember, that $\neg O p$ means that we failed to prove Op, i.e., $D \vdash -\partial^O p$. Deontic logic traditionally establishes the duality of obligation and permission ($Op \equiv \neg P \neg p$); in this view $\neg Op$ is $P \neg p$. Legal theory and legal philosophy [26] often distinguish two types of permission: *weak permission* (something is permitted because there is no contrary obligation) and *strong permission* (something is explicitly permitted as a derogation/exception to a contrary obligation). We are now in the position to analyse the reasons why there is a failure to prove Op. If there are no applicable obligation rules for p, then we can say that $\neg Op$ is a weak permission (lack of obligation), thus to contract we create a new obligation.[4] In the other case there are applicable obligation rules, but there are also applicable obligation rules for $\neg p$. Here, we have the opportunity to remove the rules that prevent the derivation of the obligation.

To conclude this section we introduce the definition of *expansion*. The main purpose of this definition is to allow us for a proper comparison with the AGM framework for belief revision advancing rational postulates relating contraction and expansion. Accordingly this definition will be used in Section 5 where we are going to carry out the analysis of the proposed contraction operations in terms of AGM. For the needs of this paper we assume that expansion is in general defined as follows.

DEFINITION 6. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $D \vdash -\#^Y p$ with $\# \in \{\Delta, \partial\}$ and $Y \in \{c, O\}$. We define $D_p^{+\#} = (F, R'^c, R'^O, \succ')$ to be the minimal* strict (defeasible) expansion theory *of D by p such that*
If $p \in \mathrm{Lit}$ or $p = Oq$

- *If $\# = \Delta$, then $R'^c \cup R'^O = R^c \cup R^O \cup \{w :\to_Y p\}$ and $\succ' = \succ$.*
- *If $\# = \partial$, then $R'^c \cup R'^O = R^c \cup R^O \cup \{w :\Rightarrow_Y p\}$ and $\succ' = \succ \cup \{w \succ r : \forall r \in R^Y[\sim p]\} \setminus \{r \succ w : r \in R^Y[\sim p]\}$.*

If $p = \neg Oq$, then $D_p^{+\#} = D_{Oq}^{-\#}$.

Before concluding this section we report a key result for expansion. Similar results for the various contraction operations are presented in Section 5 and discussed in the context of the AGM rational postulates for contraction.

THEOREM 7. *Let $D = (F, R^c, R^O, \succ)$ be any normative theory and $D \vdash -\# p$ with $\# \in \{\Delta, \partial\}$. Then $p \in \#^+(D_p^{+\#})$, unless $\# = \partial$ and $p \notin \Delta^+(D)$.*

Theorem 7 establishes the success conditions for expansion. In particular it states that one cannot use a defeasible expansion when the opposite is definitely provable.

3.2 Contraction by Adding Exceptions

Another method for contracting a legal conclusion is by adding exceptions to the rules that support this conclusion. This method is conceptually and technically different from rule removal.

REMARK 2. *At a conceptual level, the contraction of legal effects by means of adding exceptions intuitively corresponds to lawmaking acts, such as the scope derogation of statutes, or the judicial interpretation of statutes or precedents (such as distinguishing, assuming, even in this case, a rule-based representation of precedents).*
Consider, for example, the following fictional law-making act modifying art. 3 of the Italian Constitution:

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[3]This is possible only if it has been given as a fact (and to contract it we have to remove it from the facts) or there are not applicable strict obligation rules for it.

[4]A typical case of contraction of weak permission is when a legislator interprets this lack of obligation as a legal gap and so introduces new norms to fill it.

Article 3
(1) All citizens have equal social status and are equal before the law [. . .].
(2) In derogation to the provision above under paragraph 1, the citizens who are resident in Bologna may have a different social status.

This is derogation modifying the norm scope of original art. 3: in fact, it introduces an exception, derogating from the original prohibition to discriminate citizens.

Given any defeasible rule r supporting a conclusion p, exceptions of r can be modeled in DL either by adding other conflicting defeasible rules, or by adding defeaters. In this paper, we will adopt the latter strategy, since defeaters, if added, *precisely* operate in such a way that the conclusion p, without positively asserting $\neg p$, no longer holds.

If p is the conclusion to be contracted, defeaters can be added and employed for this purpose in different ways:

- they block the conclusion p in all possible cases;
- they block the conclusion p but only in specific contexts, such as typically the one where all facts needed to prove p are the case;
- they block the conclusion p by minimizing the changes over rules for p that are not in fact employed to prove p in the specific normative theory.

The first type of exception is represented by a defeater with empty body. Notice that such operation is possible iff the derivation of the literal we want to contract is not strict (this requirement holds for Definitions 9 and 10 as well).

DEFINITION 8. *Let $D = (F, R^c, R^{O}, \succ)$ be a normative theory, $D \vdash +\partial^Y p$ with $Y \in \{c, O\}$. We define $D_p^{(dft, \emptyset)} = (F, R^{c'}, R^{O'}, \succ)$ to be the minimal empty defeater contraction theory of D by p such that*

$$R^{c'} \cup R^{O'} = R^c \cup R^O \cup \{r' : \leadsto_Y \sim p\}.$$

This kind of modification is very effective but, in many cases it looks too strong, since requires the introduction of a defeater with empty body. Indeed, the activation of such a rule should depend upon the same facts needed to prove the literal to contract.

DEFINITION 9. *Let $D = (F, R^c, R^{O}, \succ)$ be a normative theory, $D \vdash +\partial^Y p$ with $Y \in \{c, O\}$, and $\mathscr{AC}[p]$ be the set of all ∂^Y active chains in D for p. We define $D_p^{(dft, F')} = (F, R'^c, R^{O}, \succ)$ to be the minimal factual defeater contraction theory of D by p, such that*

$$R^{c'} \cup R^{O'} = R^c \cup R^O \cup \{r' : f_1, \ldots, f_n \leadsto_Y \sim p\}$$

where $F' = \{f_1, \ldots, f_n\} = \bigcup_{\mathscr{C} \in \mathscr{AC}[p]} root(\mathscr{C})$.

Clearly, the contraction operations under Definitions 8 and 9 are equivalent whenever $\bigcup_{\mathscr{C} \in \mathscr{AC}[p]} root(\mathscr{C}) = \emptyset$, which happens when facts play no role in proving p from D, but the chains are triggered by rules with an empty antecedent.

The *factual defeater contraction* operation is closely related to that of refinement proposed by [22] and the modifying approach of [8] proposed for classical logic. They key idea of these is to identify the formulas (rules) "responsible" for the conclusion to be contracted and the facts triggering the operation. Then, the negation of the facts is "added" to (antecedent of) the formulas. Thus the refinement/modification of the classical formula $r = a \rightarrow b$ with facts f_1 and f_2 concluding b, is the formula $r' = a \land (\neg f_1 \lor \neg f_2) \rightarrow b$. This prevents r' and f_1, f_2 to conclude b.

DEFINITION 10. *Let $D = (F, R^c, R^{O}, \succ)$ be a normative theory, $D \vdash +\partial^Y p$ with $Y \in \{c, O\}$. We define $D_p^{(dft, \succ)} = (F, R'^c, R'^{O}, \succ')$*

to be the minimal superiority defeater contraction theory of D by p, such that

$$R^{c'} \cup R^{O'} = R^c \cup R^O \cup \{r' : \leadsto_Y \sim p\}$$
$$\succ' = \succ \cup \{s \succ r' : s \in R^Y[p] \text{ and } A(s) \cap \partial^-(D) \neq \emptyset\}.$$

This last operation minimises the impact of the added defeaters over inactive reasoning chains. Consider the following example.

EXAMPLE 2. *Suppose we have a theory containing the following piece of knowledge*

$$F = \{Resident, Italy\}$$
$$R^O = \{r_1 : Italy, Resident \Rightarrow_O PayTaxes$$
$$r_2 : Italy, Citizen \Rightarrow_O PayTaxes\}$$

and suppose we want to contract the obligation to pay taxes for the residents. If we add a defeater attacking r_1 with an empty body and which is not weaker than r_2, then we potentially prevent the theory to derive the same obligation for citizens.

We proposed two alternative contextual contraction operations based on defeaters (Definitions 9 and 10). While they are equivalent for the theory they are defined on, i.e., $E^{D_p^{(dft, F)}}) = E^{D_p^{(dft, \succ)}}$, they have a different import when we reuse the revised set of rules with a set of facts different from the facts in the theory from which we obtained the contraction.[5] The difference between the two contractions is that Definition 9 operates on all reasons for p as a single block without affecting the single individual rules, while the operation in Definition 10 blocks the single individual rules. Consider the following example:

EXAMPLE 3. *Let $D = (\{a, b\}, \emptyset, R^O, \emptyset)$ where R^O is*

$$r_1 : a \Rightarrow_O d$$
$$r_2 : b \Rightarrow_O d$$
$$r_3 : c \Rightarrow_O d$$

In $D_d^{(dft, F)}$ we have $r_4 : a, b \leadsto_O \neg d$ and $r_3 \succ r_4$. In $D_d^{(dft, \succ)}$ we have $r_4 : \leadsto_O \neg d$ and again $r_3 \succ r_4$. Consider a theory D' with the same rules and superiority relation as $D_d^{(dft, F)}$, and $F' = \{a\}$. Now $D' \vdash +\partial_O d$. Take now D'', with the same rules and superiority of $D_p^{(dft, \succ)}$, and F' as its set of facts. Here $D'' \vdash -\partial^O p$.

3.3 Contraction by Changing Rule Priorities

Another option is to delete conclusions by changing rule priorities which are used to solve rule conflicts in the normative theories.

REMARK 3. *A peculiarity of legal norms is that they may have a different ranking in the system, thus having different positions in the hierarchy of the sources of law. For example, constitutional provisions are superior to statutes, etc. Rule priorities are an immediate way for capturing this intuition. Indeed, if two norms r and r' support respectively p and $\sim p$, but r is superior to r', then r is in the logic stronger than r'. Hence, if our aim is to contract p, one possible way is to remove that priority or to modify it. In legal terms, this operation may intuitively correspond to different scenarios.*

In the law-making perspective, we may imagine the legislator which, given the inefficacy of a certain provision n due to another conflicting and superior provision n', enacts again the same n but following a new procedure that confers to it a higher status.

Consider the example discussed in Remark 1. The Italian Constitutional Court declared illegitimate art. 1 of the Legislative Act n.

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[5]If we assume that the rules represent norms in a normative system, then the facts define a case. Thus the two operations impact on how cases successive to the revision are evaluated.

124, 23 July 2008, because it was, among other things, in conflict with art. 3 of the Constitution. However, were it possible to derogate (under the same conditions of Legislative Act n. 124) from that constitutional principle, the revision would be possible only if enacted as a provision of constitutional ranking. One may argue that this change is not simply a revision of the rule priority, as it consists in: (i) adding one rule, (ii) removing it, and (iii) re-adding it with a different priority that makes it stronger. On the other hand, one could maintain that, at a more abstract level, the procedure above can be reconstructed as nothing but a change on the rule priorities.

In any case, other—and perhaps less debatable—cases can be found in the judicial practice, especially when the changes affect counts-as conclusions. Suppose we want to contract the conclusion p: if the critical rules directly supporting it are authoritative we do not have any power to remove or add new rules (such as in many civil law systems), it is still possible to work on the interpretation of existing rules by revising the superiority relation that establishes the relative strength of rules; in this way, we can still block p without changing the rule set. See Example 4 below for a real-life case illustrating the intuition.

Besides prescriptive norms (i.e., rules for obligations), there are situations where it is not allowed to revise even the set of counts-as rules (at least by courts of civil law systems), for example, whenever such counts-as rules are explicit norms stated by the lawmaker. Indeed, legal rules may refer to

1. ordinary concepts used in the law which are characterised by the counts-as rules of ordinary language (changes are always possible);
2. legal concepts that are authoritatively defined by the legal system (explicit legal definitions, case law, etc.), such as when such legal concepts
   (a) do not have any ordinary understanding: e.g., *adverse possession*;
   (b) may have both a legal and an ordinary understanding: e.g., *property*.

Prima-facie conflicts appear in legal systems for a few main reasons, such as (i) norms from different sources, (ii) norms enacted at different times, and (iii) exceptions. Accordingly, conflict resolution can be afforded by using *legal principles*. In Example 4, we show how these principles apply to handle exceptions.

EXAMPLE 4. *A couple can have offspring but, since both mother and father are affected of cystic fibrosis, they know that every their child will be affected by the same genetic anomaly. Since they want their offspring to be healthy, they request for medically assisted reproduction techniques. Their case is disputed in Court where the judge has to establish which provision prevails between art. 4 of Italian Legislative Act 40/2004[6] (r_0 and r_1) and the standard common medical practice (r_3) in force in 15 countries of the EU. The judge argues in favour of r_1 based on* lex superior *and refuses their request.*

$$R^O = \{r_0 : \neg CandidateInVitroFert \Rightarrow_O \neg Techniques\}$$
$$R^c = \{r_1 : \neg Sterility \Rightarrow_c \neg CandidateInVitroFert,$$
$$r_2 : Embryo \Rightarrow_c \neg Sterility,$$
$$r_3 : \neg Sterility, GenAnomalies \leadsto_c CandidateInVitroFert,$$
$$r_4 : \neg Sterility, GenAnomalies \Rightarrow_c \neg Healthy\}$$
$$\succ = \{r_1 \succ r_3\}.$$

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[6]"The recourse to medically assisted reproduction techniques is allowed only [. . .] in the cases of sterility".

The couple appeals to the European Court for Human Rights. The Court establishes that not permitting the medical techniques would demote the goal of family health promoted by Article 8 of the Convention. In our example, r_3 promotes the goal of family health, and thus we invert the priority between r_1 and r_3 based both on lex superior *and* lex specialis.

The study of how to change a defeasible theory by modifying the superiority relation only has been initiated by [16]. Herein we augment the framework to capture the concept of obligation, both in the positive and negative form (Ol, $\neg Ol$), and we also discuss the suitability to normative reasoning.

When we deal with changing a theory by modifying the superiority relation only, we must switch perspective with respect to the operations of *rule removal* and *defeater introduction*. The operations cannot anymore focus only on the active/inactive chains for the literal we want to change, but they must take into account the entire theory. Consider Example 5 restricted to rules from r_4 to r_8. It is not possible to act directly on the last rule for p (i.e., r_5), and introducing $r_7 \succ r_4$ will not give any improvement since r_7 is not applicable (the antecedent c is refuted). To contract p, r_6 has to be stronger than r_8. This, essentially, corresponds to "expand" D by c.

Therefore, we can state these two propositions. A contraction/expansion operation modifying only (i) the superiority relation, (ii) tuples where one of the element is a rule for p, is successful iff

Contraction: there exists an applicable rule for $\sim p$;
Expansion: there exists an applicable rule for p.

PROPOSITION 11. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $p \in \partial^+(D)$, then $D' = (F, R^c, R^O, \succ')$ such that $(\succ' \setminus \succ) \cup (\succ \setminus \succ') \subseteq \{(t, w) : C(t) = p \text{ or } C(w) = p\}$ and $p \in \partial^-(D')$ exists iff $\exists s \in R^c[\sim p].\forall a \in A(s).a \in \partial^+(D)$.*

PROPOSITION 12. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $p \in \partial^-(D)$, then $D' = (F, R^c, R^O, \succ')$ such that $(\succ' \setminus \succ) \cup (\succ \setminus \succ') \subseteq \{(t, w) : C(t) = p \text{ or } C(w) = p\}$ and $p \in \partial^+(D')$ exists iff $\exists r \in R^c[p].\forall a \in A(r).a \in \partial^+(D)$.*

Such conditions restrain too much the scope of the operation, insofar as they exclude cases where revision can successfully operate not necessarily on the last step of reasoning chains.

Accordingly, we will consider contraction and expansion on superiority relation when we are free to operate on any element of a theory.

DEFINITION 13. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $p \in \partial^+(D)$. We define $D_p^{(\succ, -)} = (F, R^c, R^O, \succ^-)$ to be the superiority contraction theory of D by p, where $\succ^-$ is a minimal superiority relation such that $p \in \partial^-(D_p^{(\succ, -)})$.*

DEFINITION 14. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $p \in \partial^-(D)$. We define $D_p^{(\succ, +)} = (F, R^c, R^O, \succ^+)$ to be the superiority expansion theory of D by p, where $\succ^+$ is a minimal superiority relation such that $p \in \partial^+(D_p^{(\succ, +)})$.*

It is not possible to give a simple procedural definition neither for contraction, nor for expansion. Indeed, as stated before, the superiority-contraction operation on p in general does not involve only active chains for p (by removing some tuples from the superiority relation) but it may be necessary to modify other chains by expanding some other literals (see Example 5). Moreover, contracting a negative obligation ($p = \neg Ol$) is feasible only in a "indirect" way since our framework does not allow rules for it and its derivation depends upon the refutability of Ol. Accordingly, we must force l to be mandatory in the theory, and this results in expanding the theory by Ol.

Analogous reasonings hold for the expansion operation.

EXAMPLE 5. *Let $D = (\emptyset, R^c, \emptyset, \{(r_2, r_1)\})$, where R^c is*

$$\Rightarrow_{r_1} \quad \neg a$$
$$\Rightarrow_{r_2} \quad a \quad \Rightarrow_{r_3} \quad p$$
$$\Rightarrow_{r_4} \quad b \quad \Rightarrow_{r_5} \quad p$$
$$\Rightarrow_{r_6} \quad c \quad \Rightarrow_{r_7} \quad \neg b$$
$$\Rightarrow_{r_8} \quad \neg c.$$

There are two chains proving p, namely: $\mathscr{C}_1 = \{r_2\}_1^1 \{r_3\}_1^2$ and $\mathscr{C}_1 = \{r_3\}_2^1 \{r_5\}_2^2$. We block $\mathscr{C}_1$ by removing tuple (r_2, r_1), while we block $\mathscr{C}_2$ by "expanding" literal c with (r_6, r_8). Since the final relation is minimal, there is no need to add (r_7, r_4).

[17] points out that two main problematics affect defeasible contraction and expansion by only changing the superiority relation: (i) *tautologicity* of a literal, i.e., a literal that is true in every interpretation; (ii) *∂-unreachability* of a literal, i.e., a literal depending on a literal and its complementary.[7] We refer the interested reader to [17] for a formal characterisation of the topic. Clearly, these two literal states prevent the successfulness of the operations.

THEOREM 15. *Let $D = (F, R^c, R^O, \succ)$ be any normative theory. Then $p \in \partial^+(D_p^{(\succ, +)})$, unless either p is ∂-unreachable or there is no chain supporting p.*

Proof Sketch. If p is ∂-unreachable, we show that p can be proved only if the theory D is inconsistent. Let us suppose a counts-as chain $\mathscr{C}$ supports p. Since p is ∂-unreachable by hypothesis, then there exists a literal q in $\mathscr{C}$ (q may be p itself) that depends on a literal a and $\neg a$. Then, in order to prove q, D must prove both a and $\neg a$. Hence, D is inconsistent.

We can assume that D is consistent. If there is no chain supporting p, then the only way to prove it is by introducing new rules in R. On the contrary, if there is a chain $\mathscr{C}$ supporting p, the superiority relation $\succ'$ where each literal in $\mathscr{C}$ wins is such that $D' = (F, R^c, R^O, \succ') \vdash +\partial^c p$.

The analogous result of success for superiority contraction is postponed to the AGM postulate analysis of Section 5.

4. BEYOND CONTRACTION

The previous sections proposed different contraction operations. They offer distinct and, in general, non-equivalent options for manipulating the sets of norms in order to delete unwanted legal effects. One general assumption for each operation is that it is legal permitted and that we are empowered to perform it: in fact, a peculiar aspect of the law is that it claims to regulate its own changes.

Bearing in mind this preliminary consideration, let us consider the following problems:

1. *Strict rules and conclusions*: If a conclusion p is strict (i.e., it has been obtained using only facts and strict rules), then adding exceptions is an ineffective option, thus rule removal is the only successful measure. The same holds even when p is a defeasible conclusion obtained by applying a strict rule, which means that at least one of its antecedents has been defeasibly proved. However, suppose you have some authoritative provisions that cannot be removed (such as some constitutional provisions), or you do not have any power or are not sufficiently authoritative (e.g., the existing rules are case law but the court has a lower ranking with respect to the court which produced those rules), or, finally, that you simply do not want to remove the rule supporting p because you think it would be meaningless to delete that norm. What to do?

[7]A literal *a depends* on literal *b* iff (1) $b = a$ or (2) $\forall r \in R[a]$, either (2.1) $b \in A(r)$, or (2.2) $\exists c$ such that $c \in A(r)$, and c depends on *b*.

2. *Deleting indirect conclusions*: The procedures presented in Section 3 do not address the problems under point 1 above and, in general, do not cover the case where the contraction of the literal p is not made by directly attacking or removing the rules supporting it, but by attacking other "links" in the reasoning chains leading to trigger those rules for p. This option can help us avoid difficulties due to our lack of power. Also, deleting indirect conclusions can be the best option when it offers successful strategies that are also meaningful from the legal viewpoint (see Remark 4).

As we argued above, rule removal is needed when the contracted literal is a strict conclusion or the rules that should be blocked are strict. Definitions 4 and 5 present two rule-removal operations for the successful contraction of strict and defeasible conclusions, respectively. However, consider the following example:

EXAMPLE 6. *Consider a normative theory D which contains $\{a, b\}$ as the set of facts and the following set of counts-as rules:*

$$R^c = \{r_1 : a \rightarrow_c \neg c, \; r_2 : d \Rightarrow_c c, \; r_3 : b \rightarrow_c d, r_4 : d, a \rightarrow_c \neg c\}$$

Suppose we want to contract $\neg c$, which is in Δ^+ of the extension of D. In this theory we have only the following two strict counts-as reasoning chains for $\neg c$:

$$\mathscr{C}^1 = \{r_1 : a \rightarrow_c \neg c\}_1^1$$
$$\mathscr{C}_2 = \{r_3 : b \rightarrow_c d\}_2^1, \{r_4 : d, a \rightarrow_c \neg c\}_2^2$$

Since the two chains do not share any rule, we have to remove at least two rules. An option is the removal of r_1 and r_4, but also the removal of r_1 and r_3 would work. This latter option is not covered by Definition 4.

The previous example allows us to introduce the second question:

REMARK 4. *Suppose we have the regulative legal rule*

$$r_1 : Homicide \Rightarrow_O Prison,$$

and the following set of counts-as rules defining the concept of homicide:

$$R^c = \{r_2 : Embryo \rightarrow_c AliveHuman, \; r_3 : AliveHuman \rightarrow_c Person,$$
$$r_4 : Person, Kill \rightarrow_c Homicide.\}$$

Suppose a homicide is when someone kills a person, which is an alive human subject. Also, killing an embryo falls within the scope of homicide, because an embryo is considered, too, as an alive human subject. If we deny that killing an embryo amounts to a homicide, we have three options: removing r_2, r_3, or r_4. However, it would be better to remove r_2 rather than r_3 or r_4: the second option would no longer allow to establish in general when a subject is a person, while the the third would deny that killing a person is a homicide, thus preventing the general application of r_1.

Notice that the same argument presented in Example 4 can re-iterated for any defeasible conclusion. For instance, just replace rules r_1-r_4 with defeasible rules with the same bodies and heads and suppose to add defeaters. You can attack either r_2, or r_3, or r_4, but, again, blocking r_2 looks conceptually more acceptable.

Hence, we need a way for deleting conclusions by working on other, indirect conclusions in the active chains. A general procedure that successfully extends all contraction operations described in Section 3 is the following:

DEFINITION 16. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory, $D \vdash +\partial^Y p$, and $(\cdot)_S^-$ be one of the contraction operation defined in Section 3. We define D_p^* to be the* iterated contraction *theory of D by p where: (i) $D^0 = D$, (ii) $\mathscr{AC}^i[p] = \{\mathscr{C}_1^i, \ldots, \mathscr{C}_n^i\}$ is the set of all*

the ∂^Y active chains in D^i for p, and (iii) $q_1^i, \ldots, q_m^i$ is a minimal set of literals in $\mathscr{A}\mathscr{C}^i$ such that $\forall \mathscr{C}_k^i \in \mathscr{A}\mathscr{C}^i[p]$, $\mathscr{C}_k^i \notin \mathscr{A}\mathscr{C}^{i+1}[p]$. Then,

$$D^{i+1} = \begin{cases} D^i & \text{if } \mathscr{A}\mathscr{C}^i[p] = \emptyset; \\ (D^i)^-_{\{q_1^i,\ldots,q_m^i\}} & \text{otherwise} \end{cases}$$

When the actions to be undertaken must work on different rules than the ones with head p, we need a procedure to make inactive all the active chains for p. To this end, we can apply methodologies proposed in Definitions 4–5 or 8–10 (depending on the revision strategy one has in mind) to a minimal set of elements $(q_1^i, \ldots, q_m^i)$ such that each element belongs at least to one active chain. Such a "deactivating" procedure may, in turn, trigger other previously not applicable rules in inactive chains for p, thus generating a new set of active chains. Consequently, Definition 16 must be iterative and performs such operations until no active chains for p exist.

REMARK 5. *The need in Definition 16 of a fix-point construction depends on the fact that, for example, the removal of p from ∂_D^+, which does not affect the rules in $R_d^c[p]$, must take into account that blocking some other literals that are used to argue in favour of p may trigger in turn some rules proving p, like in the theory $D = (\{a,d\}, R^c, \emptyset, \emptyset)$, where R^c is*

$$
\begin{array}{llll}
a & \Rightarrow_{r_1} & b & \\
 & & b & \Rightarrow_{r_2} & p \\
 & & b & \Rightarrow_{r_3} & c \\
 & & d & \Rightarrow_{r_4} & \neg c & \Rightarrow_{r_5} & p.
\end{array}
$$

At the first step, the only active chain is $\mathscr{C}_1 = \{r_1\}_1^1 \{r_2\}_1^2 \{r_3\}_1^3$. Let us examine two different scenarios: (1.) we use the defeasible removal contraction, (2.) factual defeater contraction.

1. *Removing r_1 makes r_4 not applicable, as well as the chain $\mathscr{C}_2 = \{r_4\}_2^1 \{r_5\}_2^2$ active. Hence, r_4 is removed.*
2. *Defeaters $r' : a \rightsquigarrow \sim b$ and $r'' : d \rightsquigarrow c$ are added to the theory, during the first, and second step, respectively.*

5. AGM POSTULATES ANALYSIS

Throughout this section, we analyse the AGM *contraction* postulates applied to operations defined in Section 3 only, and we do not consider the iterated contraction of Section 4. This is motivated by the fact that AGM contraction and iterated contraction tackle the problem from different perspectives: the first is to remove a belief and all its consequences, the latter is to remove *something* that implies such a belief.

In the remainder we assume that the reader is familiar with the terminology used in the AGM framework, in particular with the notions of *belief* and *belief set*.[8] Specifically to our framework, the concept of belief set is represented by the concept of extension of a defeasible theory. Given a theory D and $\# \in \{\Delta, \partial\}$, we state that when a literal p is believed ($p \in K$ in AGM notation) then $p \in \#^+(D)$. Conversely, if p is not believed (i.e., $p \notin K$) then $p \in \#^-(D)$.[9]

We use the shortcut notation D_p^- to denote the theory resulting from one of the contraction operations applied to a normative theory $D = (R^c, R^O, \succ)$ for a literal $p \in \#^+(D)$ among $D_p^{-\#}$, $D_p^{(dft,*)}$, and $D_p^{(\succ,-)}$. First, each postulate is briefly explained and formulated

to join our framework; it follows the analysis for each operation: **Rem.**, **Dft.** and **Sup.**, respectively.

$(K \dot{-} 1)$ The first postulate states that when a belief set is contracted by a sentence, the outcome should be logically closed. While in classical logic there is no difference from contracting a theory or its base, in DL this is not the case since there is a distinction between a theory (i.e., a set of rules) and its extension (i.e., its set of conclusions) Thus, for a given extension in DL, there are multiple (possibly not equivalent) theories that generate it.

D_p^- is a theory. $\hspace{3em} (K \dot{-} 1)$

Rem., **Dft.**: the postulate trivially holds.
Sup.: the operation itself must not create a cycle within the superiority relation. This is guaranteed by the following proposition.

PROPOSITION 17. *Let $D = (F, R^c, R^O, \succ)$ be a normative theory and $D \vdash +\partial^Y p$. Then $D^{(\succ,-)} = (F, R'^c, R'^O, \succ')$ is such that $\succ'$ is acyclic.*

Proof Sketch. Let us proceed by case inspection on p. If $p \in \text{Lit}$ or $p = Oq$, let us suppose, by contradiction, that there is a cycle in $\succ'$. Since $\succ'$ is minimal, it can be obtained from $\succ$ by simply removing tuples from the superiority relation. Then each element of $\succ'$ is an element of $\succ$ and the cycle in $\succ'$ is also in $\succ$, against the hypothesis. Otherwise, $p = \neg Oq$ and we need to expand D by Oq. Theorem 15 states conditions on Oq to have the thesis. $\hspace{1em} \square$

$(K \dot{-} 2)$ A contraction operation always produces a belief set smaller than the original. While AGM focuses only on "positive" beliefs, we have to study four types of conclusions ($\Delta^\pm, \partial^\pm$). Thus, the set of formulas believed should be smaller, while the set of formulas we do not believe should be increased by the formula we contract.

$\#^+(D_p^-) \subseteq \#^+(D)$ and $\#^-(D_p^-) \supseteq \#^-(D)$. $\hspace{1em} (K \dot{-} 2)$

In general, this postulate fails in any sceptical non-monotonic formalism. Suppose that we know a and we have the rules $\Rightarrow_c p$ and $a \Rightarrow_c \neg p$. Then a is sceptically provable, and p is not. But if we decide to contract a, then p becomes defeasibly provable, thus we have $p \in \partial^-(D)$ but $p \in \partial^+(D_p^-)$.

$(K \dot{-} 3)$ If the belief to be contracted is not in the initial belief set, the operation does not change the initial theory.

If $p \in \#^-(D)$ then $\#^\pm(D_p^-) = \#^\pm(D)$. $\hspace{1em} (K \dot{-} 3)$

The postulate trivially holds.

$(K \dot{-} 4)$ The fourth AGM postulate states that the only literals that are immutable in the contraction process are tautologies.

If $p \in \#^+(D_p^-)$ then p is a tautology. $\hspace{1em} (K \dot{-} 4)$

The concept of tautology is that of a statement that is true in every interpretation, and as such it cannot be contracted. Depending on the particular contraction operation, p is a tautology if

Rem. $p \in F$ when $\# = \Delta$, or $p \in \Delta^+(D)$ otherwise;
Dft. $p \in \Delta^+(D)$.

Sup. When we deal only with the superiority relation, formalising the notion of tautology is an NP-complete problem. We have three possible cases:

1. $p \in \Delta^+(D)$.
2. There exists an active chain made of elements for which there are no rules for the opposite, like chain $\mathscr{C}_1[p] = \{r_1\}_1^1, \{r_2\}_1^2$ in the following theory:

$$D = (\{a\}, R^c, \emptyset, \emptyset), \text{ where } R^c \text{ is}$$

$$a \quad \Rightarrow_{r_1} \quad b \quad \Rightarrow_{r_2} \quad p$$
$$\Rightarrow_{r_3} \quad c \quad \Rightarrow_{r_4} \quad p$$
$$\Rightarrow_{r_5} \quad \neg c.$$

3. There exists a couple of active chains $\mathscr{C}_1$ and $\mathscr{C}_2$ for p that cannot be blocked at the same time. All the elements of $\mathscr{C}_1$ and $\mathscr{C}_2$ satisfy: conditions (1.), or condition (2.), or given $c_1 \in \mathscr{C}_1$ there exists $c_2 \in \mathscr{C}_2$ such that $\neg c_1$ depends on a literal l and $\neg c_2$ depends on $\neg l$, as in the following example.

EXAMPLE 7. *Let $D = (\emptyset, R^c, \emptyset, \emptyset)$, where R^c is*

$$\Rightarrow_{r_1} \quad l \quad \Rightarrow_{r_2} \quad \neg a$$
$$\Rightarrow_{r_3} \quad a \quad \Rightarrow_{r_4} \quad p$$
$$\Rightarrow_{r_5} \quad b \quad \Rightarrow_{r_6} \quad p$$
$$\Rightarrow_{r_7} \quad \neg l \quad \Rightarrow_{r_8} \quad \neg b.$$

The NP-hardness of the problem relies on the fact that "strategically" combining two of the patterns of Example 7 may result in a non-tautological literal (a complete discussion of the topic is in [17]). Therefore, the problem of inspecting the whole theory in order to find if those structures lead to a tautology is NP-complete.

THEOREM 18. *Let $D = (F, R^c, R^O, \succ)$ be any normative theory. Then $p \in \partial^-(D_p^-)$, unless p is a tautology.*

Proof sketch. (**Case Rem.**) If $p \notin F$, then there exist a strict rule proving it. Analogously, if $p \notin \Delta^+(D)$, the rule proving it is defeasible. Strict and defeasible removal contraction erase all (strict or defeasible, respectively) applicable rules for p, when $p \in \mathrm{Lit}$ or $p = Oq$; otherwise, they add a strict or stronger defeasible rule for Oq. In both cases, p cannot be derived once the operation applies.

(**Case Dft.**) All three defeater contractions add a defeater which is not weaker than the last rule in the active chain for p. If $p \in \mathrm{Lit}$ or $p = Oq$, then such a defeater is against p, otherwise is against Oq. In both cases, the new defeater is effective iff $p \notin F$ or if there does not exist a strict derivation for it.

(**Case Sup.**) Let us suppose p is not a tautology as previously defined for the case of superiority contraction. Then, there exists an assignment of the superiority relation such for every active chain for p there exists a rule applicable in D but that is not in $D_p^{(\succ, -)}$.

$(K\dot-5)$ The fifth AGM postulate states the possibility of recovery, i.e., that contracting and then expanding by the same belief returns at least the initial theory.

If $p \in \#^+(D)$ then $\#^\pm(D) \subseteq \#^\pm((D_p^-)_p^+)$. $\quad\quad\quad (K\dot-5)$

Rem., Dft. The postulate holds. The effect of expansion is moving from $\#^+(D)$ to $\#^+(D) \cup \{p\}$. Since $\Delta^+(D) \subseteq \partial^+(D)$, then $((D_p^-)_p^+) = (\#^+(D) - \{p\}) \cup \{p\} = \#^+(D)$. Notice that, if we contract vacuously then the expansion can generate a structure that is not an extension. Hence, we have to assume that $p \in \#^+(D)$.

Sup. This postulate cannot be adopted since the backward step may involve different superiority tuples than the ones which have been contracted, as the following example shows.

EXAMPLE 8. *Let $D = (\emptyset, R^c, \emptyset, \{(r_1, r_3)\})$, where R^c is*

$$\Rightarrow_{r_1} \quad a \quad \Rightarrow_{r_2} \quad p$$
$$\Rightarrow_{r_3} \quad \neg a$$
$$\Rightarrow_{r_4} \quad b \quad \Rightarrow_{r_5} \quad p$$
$$\Rightarrow_{r_6} \quad \neg b.$$

$D_p^{(\succ, -)}$ *is obtained by removing $r_1 \succ r_3$. We can now expand by operating on b, the new superiority relation is $\succ^+ = \{(r_4, r_6)\}$.*

Nevertheless, if all operations in the superiority contraction process can be traced, then we can easily backtrack and obtain the initial theory, satisfying the postulate.

$(K\dot-6)$ The sixth AGM postulate states that if two beliefs are logically equivalent, then contracting by either by one of them produces the same result. In DL, the language is formed by literals, thus two elements are equivalent if they represent the same literal.

If $\vdash p \equiv q$ then $\#^\pm(D_p^-) = \#^\pm(D_q^-)$. $\quad\quad\quad (K\dot-6)$

In the framework of DL, the language is restricted to literals, thus two elements p and q are equivalent only if they represent the same literal. For this reason, the sixth postulate straightforwardly follows.

$(K\dot-7 - K\dot-8)$ The seventh and the eighth postulates relate two individual contractions with respect to a pair of sentences ψ and χ, with the contraction of their conjunction $\psi \wedge \chi$. As already stated, in DL there are no logical connectives, and a conjunction of literals is equivalent to the set of the same literals; the same reasoning used to introduce postulate $(K\dot-2)$ applies here. Thus, the two postulates can be rewritten as

$\#^+(D_p^-) \cap \#^+(D_q^-) \subseteq \#^+(D_{p,q}^-)$ and $\quad\quad (K\dot-7)$
$\#^-(D_p^-) \cap \#^-(D_p^-) \supseteq \#^-(D_{p,q}^-).$

If $p \in \#^-(D_{p,q}^-)$ then $\#^+(D_{p,q}^-) \subseteq \#^+(D_p^-)$ and $\quad (K\dot-8)$
$\#^-(D_p^-) \subseteq \#^-(D_{p,q}^-).$

Rem., Dft., Sup. Postulates $(K\dot-7)$ and $(K\dot-8)$ do not hold for the same reason formulated for postulate $(K\dot-2)$: the two composite contraction AGM postulates cannot be naturally accommodated in our framework, because they contradict the sceptical nonmonotonic nature of our underlying logical machinery. For **Rem.** and **Dft.** see the discussion above on $(K\dot-2)$. The following example shows for **Sup.** the truth of the statement for both of them.

EXAMPLE 9. *Let $D = (\emptyset, R^c, \emptyset, \{(r_1, r_2), (r_3, r_4), (r_5, r_7), (r_9, r_6), (r_8, r_{10})\})$, where R^c is*

$$\Rightarrow_{r_1} \quad \neg a$$
$$\Rightarrow_{r_2} \quad a \quad \Rightarrow_{r_3} \quad c$$
$$\Rightarrow_{r_4} \quad \neg c \quad\quad \neg c, \neg d \Rightarrow_{r_5} \quad p$$
$$\Rightarrow_{r_6} \quad \neg d \quad\quad\quad \Rightarrow_{r_7} \quad \neg p$$
$$\Rightarrow_{r_8} \quad b \quad \Rightarrow_{r_9} \quad d$$
$$\Rightarrow_{r_{10}} \quad \neg b.$$

The defeasible extensions consists of $\partial^+(D) = \{a, b, c, d, \neg p\}$, and $\partial^-(D) = \{\neg a, \neg b, \neg c, \neg d, p\}$. Let us contract D by a and by b, obtaining:

- $\partial^+(D_a^{(\succ, -)}) = \{b, \neg c, d, \neg p\}$,
- $\partial^+(D_b^{(\succ, -)}) = \{a, c, \neg d, \neg p\}$,
- $\partial^-(D_a^{(\succ, -)}) = \{a, \neg a, \neg b, c, \neg d, p\}$,
- $\partial^-(D_b^{(\succ, -)}) = \{\neg a, b, \neg b, \neg c, d, p\}$.

The respectively intersections are:

$$\partial^+(D_a^{(\succ, -)}) \cap \partial^+(D_b^{(\succ, -)}) = \{\neg p\}$$
$$\partial^-(D_a^{(\succ, -)}) \cap \partial^-(D_b^{(\succ, -)}) = \{\neg a, \neg b, p\}.$$

We can now contract a and b simultaneously, and obtain

$$\partial^+(D_{a,b}^{(\succ, -)}) = \{\neg c, \neg d, p\}$$
$$\partial^-(D_{a,b}^{(\succ, -)}) = \{a, \neg a, b, \neg b, c, d, \neg p\}$$

proving our claim.

Throughout postulates $(K\dot-1)$ to $(K\dot-8)$ we took care of the transition effects of the contraction process, due to the specific nature of

positive and negative beliefs in Defeasible Logic. However, for each postulate this specificity has no effect. In fact, what can be claimed for contractions in $\#^+$ extends to $\#^-$, and vice versa.

6. CONCLUSIONS

This paper is a systematic study of legal dynamics in regard to the problem of deleting unwanted legal effects. The study was based on one expressive variant of DL able to model norms defining legal concepts (counts-as rules) and norms stating different deontic effects. We argued that legal contraction is an umbrella concept including operations that are conceptually and technically different: removing rules, adding exceptions (which breaks down into three subtypes), and modifying rule priorities. The peculiarities of deleting legal effects led us to identify an extension of those operations that work on the indirect conclusions that are needed for deriving the target effect of the contraction.

Little research efforts have been devoted to formal models of legal change. The AGM framework [1] is still the main reference framework. However, the operation of contraction for legal dynamics is in AGM perhaps the most controversial one, due to some postulates such as recovery [12, 27] and to the elusive nature of legal changes, such as derogations, repeals, abrogations and annulments [12].

Another limit of standard AGM framework is that it is very abstract and so it is hard to model the distinction between norm change and the change of normative effects (such as obligation change). This difficulty has been addressed in logical frameworks combining AGM ideas with richer rule-based logical systems, such as standard DL [23] or Input/Output Logic [7, 25]. [27] suggested a different route, i.e., employing in the law existing techniques—such as iterated belief change, two-dimensional belief change, belief bases, and weakened contraction—that can obviate problems identified in [12] for standard AGM.

A different perspective beyond AGM was offered in [13, 14, 12], where rule-based systems are combined with complex temporal structures. The resulting logics are very rich but to hard manage.

This paper works in the direction of [23, 7, 25], but presents a more expressive logic than the one of [23], and has a much more systematic view, in regard to the legal contractions, than [7, 25].

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