De Re Modal Semantics

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Any quantified modal logic with the world-relative interpretation presents important questions about the treatment of *de re* modalities when a term does not refer to an existing individual. We provide a system of quantified modal logic that overcomes them. This result is achieved, semantically, with a dual quantification (one for terms not bound by modal operators, and one for terms not modally bound) and with the help of positive and negative extension (interpretation) of predicates.

Kripke frames are extended with: a function $d : W \mapsto 2^D$ which assigns to each world its domain; $\veebar(\phi, w_i) \subseteq d(w_i)^n$ and $\barwedge(\phi, w_i) \subseteq D^n$, respectively the positive extension and the negative extension of a predicate ϕ wrt a given world; a set F of partial (ostension) functions $f : W \times \mathfrak{C} \mapsto D$. $\veebar$ and $\barwedge$ satisfy the following conditions:

1. If $\vec{d} \in \veebar(\phi, w_i)$ then $\vec{d} \notin \barwedge(\phi, w_i)$;
2. $\forall c : f(w_i, c) = d \in d(w_i)$, if $\vec{d} \notin \veebar(\phi, w_i)$ then $\vec{d} \in \barwedge(\phi, w_i)$.

The idea behind negative and positive extension of a predicate ϕ is that ϕ is believed true until a counterexample is provided, i.e. until an element does not belong to the negative extension.

Given a model M, the notion of a formula α *being true in* M *at* w_i, $\models^M_{w_i} \alpha$ contains the following clauses:

1. $\models^M_{w_i} \phi(c)$ iff (1) $f(w_i, c) \in d(w_i)$ and $f(w_i, c) \in \veebar(\phi, w_i)$, or
   (2) $f(w_i, c) \notin d(w_i)$ and $f(w_i, c) \notin \barwedge(\phi, w_i)$;
2. $\models^M_{w_i} \forall x(\gamma(x)\#\Box\delta(x))$ iff $\forall c_h, c_k, w_j$ if $w_i R w_j$, $f(w_j, c_h) \in d(w_j)$ and $f(w_j, c_h) = f(w_i, c_k)$, then $\models^M_{w_i} \gamma(c_k/x)\#\Box\delta(c_h/x)$.

In this way quantifiers establish the range of quantification with respect to a modality. Open and objective formulas are evaluated in the whole domain, whereas *de re* formulas take care only of the individuals in the valuation of actual world(s) domain(s).